c. If Australia is a continent then Tasmania is an island.
d. If horses are reptiles then the moon is not a star.
e. If plastic isn't a mineral then phones are only ever made of clay.
f. If bridge is a board game then poker is not a card game.

Question Using the supplied abbreviations, translate the following propositions into logical symbolism:

d	Dante is the greatest poet	m	Marlowe died in a brawl
o	O'Hara was run over by a dune-buggy	b	The best poets die young
s	Shakespeare is the greatest poet	c	Coleridge died happy

(i) If the best poets die young then Coleridge died happy.
(ii) If Dante is the greatest poet then Shakespeare isn't.
(iii) If Marlowe didn't die in a brawl then O'Hara was run over by a dune-buggy.
(iv) If Coleridge didn't die happy then either Shakespeare or Dante is the greatest poet.
(v) If O'Hara wasn't run over by a dune-buggy then either Dante is not the greatest poet or if Marlowe died in a brawl then the best poets die young.

6.3 Logic as representation and perfection of meaning

Our exposition of the propositional connectives *and*, *or*, *not* and *if…then* has revealed that their truth-functional definitions are quite often counter-intuitive and unnatural, failing to correspond to the norms of ordinary English. None of the operators corresponds perfectly with any English equivalent (see Bach 2002 for further discussion). The discontinuity between natural language *and* and & has already been discussed in Chapter 4 (see 4.3.1); another example of the discontinuity between natural language and logical operators is provided by negation: given principles which we have not made fully explicit here but which are reasonably obvious, two negatives cancel each other out, giving a positive statement. Thus, the proposition $\neg \neg p$ is logically equivalent to p. This logical principle is well understood by educated speakers of English, who regularly avoid the use of double negatives like those in (24):

(24) *He didn't say nothing.*
Are you going to spend your whole life not trusting nobody?
Nobody here didn't point no gun at nobody (Huddleston and Pullum 2002: 846, adapted)
It ain't no way no girl can't wear no platforms to no amusement park (Baugh 1983: 83, cited in Martínez 2003: 480)

Constructions like this were once common in English; their decline only started in the seventeenth century (Martínez 2003: 478). The prescriptive grammatical tradition of English has proscribed the use of such double negatives for hundreds of years; nevertheless, the double negative continues to thrive 'as a regular and widespread feature of non-standard dialects of English across the world' (Huddleston and Pullum 2002: 847). Furthermore, in many languages, such as Spanish (25a), Italian (25b), Portuguese (25c) and Ancient Greek (25d), double negatives regularly perform a reinforcing, rather than a cancelling function:

(25) a. *No* *vino* *nadie*
not came no one
'No one came' (Martínez 2003: 477)

b. *Giovanni* *non* *vide* *nessuno*
Giovanni not saw no one
'Giovanni didn't see anyone' (Martínez 2003: 477)

c. *Não* *viste* *nada?*
not saw nothing
'Didn't you see anything? (Martínez 2003: 477)

d. *ouk* *ara…* *gignōsketai* *tōn* *eidōn* *ouden*
not then is known of the forms nothing
'Of the forms then nothing is known' (Plato, *Parmenides* 134b, cited by Horrocks 1997: 274)

Another particularly flagrant example of discontinuity between the operators and natural language is provided by the material conditional; indeed, the correspondence between ⊃ and ordinary language has been a matter of philosophical controversy since the time of Stoic logicians in antiquity. Case (d) of the truth table is the most problematic, since it means that a statement is automatically true where the antecedent is false and the consequent is true. But this seems to fly in the face of our intuitions about ordinary language. To borrow Girle's example (2002: 240), why should it be automatically true that *If Henry VIII was a bachelor then he was King of England*? As Girle comments (2002: 240), many people 'would want to say that it's very difficult to say whether it's true or false. To say it's automatically true is too much.' The truth-functional definition of ⊃ therefore seems not at all accurate as a representation of the meaning of English *if…then*. This is not a peculiarity of English: conditional expressions in other languages seem to be like English, and unlike ⊃, in this respect.

We will see more examples of discrepancies between logic and ordinary language later in the chapter, and logicians have expended considerable effort to reconcile the two. The theory of conversational implicature developed by Grice, discussed in 4.3, is one such attempt. This theory leaves the truth-functional definitions of the operators intact, but there have been other attempts to amend the truth tables in order to bring the meanings of the operators into line with their natural language equivalents. For reasons that go beyond the scope of this chapter, however, no one satisfactory way of doing this has ever gained wide acceptance: it would seem that we are stuck with the operators in their current state.

The clash between the meanings of the logical operators and their ordinary language equivalents reveals a contrast between two different interpretations of the nature of logic: logic as a representation and logic as a perfection of meaning. The two construals carry very different implications for the relevance of logic to linguistic semantics. According to the first view of logic, the truth-functional definitions of logical operators like ¬, &, ∨ and ⊃ represent fundamental categories of human thought, and, as such, underlie the meanings of natural language at a certain degree of abstraction. Even though actual natural languages typically do not contain words whose meanings correspond to those of the logical operators, this does not mean that the logical operators are not representative of the meanings relevant to the analysis of natural language, nor that logic as a whole has nothing to do with the study of natural language. For McCawley (1981), for example, there is no clash between logic and linguistics: the two disciplines share a subject matter: meaning. Many linguists, indeed, would maintain that discontinuities between natural language and logic like those discussed in this section are to be explained by the fact that natural languages possess a pragmatic dimension which prevents the logical operators from finding exact equivalents in ordinary discourse. The fact that logical notions like ¬, &, ∨ and ⊃ are not transparently reflected in natural language is in itself no reason to doubt their importance as fundamental primitives of meaning, any more than the fact that people cannot draw freehand circles means that we do not have a concept CIRCLE. 'Formal' semantic theories in linguistics assume precisely that the principles of logic form part of a viable model of natural language meaning.

According to the second view of the relation of logic to natural language, logic does not distil principles already present in natural language, but transcends and perfects natural language. While logical principles may reveal the fundamental workings of thought, their utility lies precisely in that they allow us to escape the inadequacies of ordinary language. For Grice (1989), the fact that discrepancies exist between logical operators and their natural language equivalents 'is to be regarded as an imperfection of natural languages': the natural language expressions corresponding (imperfectly) to the logical operators 'cannot be regarded as finally acceptable, and may turn out to be, finally, not fully intelligible' (1989: 23). Natural language is not, therefore, to be appealed to in logical investigation, and the validity of logic has nothing to do with whether it turns out to be useful as a representation of natural language meaning.

This second view is appealing to logicians who see the principal purpose of logic as being to provide a solid basis for accurate reasoning of the sort required by science. Wittgenstein sums up this point of view when he says that 'the crystalline purity of logic was of course not a result of investigation; it was a requirement' (1953: §107): in other words, the value of logic is precisely that it takes us beyond the imperfections of natural language, allowing us to discern logical structures which the messiness of natural language obscures. As Barwise and Perry comment (1983: 28), the principal concern of the founders of modern logic – Frege, Russell and Whitehead, Gödel, and Tarski – was to provide a sure footing for the study of mathematics, and hence of science. This meant that logical investigation was in fact often oriented away from natural language, embodying assumptions designed to put mathematical notions on a sound footing, which have made it 'increasingly difficult to adapt the ideas of standard model theory to the semantics of natural languages'.

We will take up this question again at the end of the chapter.

6.4 Predicate logic

Consider the following argument:

(26) 1. All primates are hairy.
 2. Koko is a primate.
 therefore
 3. Koko is hairy.

This argument is clearly valid. But notice that using the propositional symbols we have introduced so far, we cannot demonstrate this validity. The two premises and the conclusion of (26) each express different propositions. We have no way, in our existing symbolism, of showing that these propositions involve the recurrent elements *Koko*, *primate* and *hairy*. As things stand, we can only assign a different letter variable to each of the propositions, giving us the following symbolism for the argument:

(27) 1. All primates are hairy.	*p*
2. Koko is a primate.	*q*
therefore	
3. Koko is hairy.	*r*

The logical form '*p*, *q*, therefore *r*' is thus the only way we have in propositional logic to symbolize the structure of the argument. But, in itself, this logical form is invalid. To see this, recall that *p*, *q*, and *r* can refer to *any* proposition; thus (28) is equally an instance of an argument with the form *p*, *q*, therefore *r*:

(28) 1. Henry Darger created a beautiful and violent fantasy world.	*p*
2. India is smaller than Africa.	*q*
therefore	
3. Thinking is the soul's conversation with itself.	*r*

Clearly, wherever the validity of (27) comes from, it does not derive from its conformity to the logical form *p*, *q* therefore *r*; as demonstrated by (28), not all arguments of this form are valid. Instead, the validity of (27) springs principally from the meaning of the term *all*. In order to symbolize (27) in a way that makes its validity clear, we will need to go beyond a purely propositional notation so that the idea of 'all' can be captured in a logically rigorous way.

Now consider the argument in (29):

(29) Some things can only be seen when they move;
therefore
if nothing moves, there are things which can't be seen. (Ruyer 1998: 101)

Propositionally, this argument has the form *p*, therefore $(q \supset r)$: again, a clearly invalid argument form. Yet (29) is obviously valid, and its validity derives from the meaning of the term *some*. In order to symbolize the validity of arguments like (29), we therefore also need some way of capturing the idea of 'some'.

'Some' and 'all' are the basic notions in the other branch of logic with which we will be concerned in this chapter. This branch is **predicate logic**, also known as **quantificational** or **first-order logic**. What exactly are predicates? Let's examine (27) again. From a logical point of view, (27) contains three basic types of term: terms referring to individuals, such as *Koko*, terms referring to quantities, like *all*, and general terms like *primate* and *hairy*. Terms referring to individuals are called **singular terms** or **individual constants**. We will symbolize them with lower case letters. Koko, for instance, can be symbolized simply by *k*. Terms referring to quantities like 'all' or 'some' are called quantifiers: we will introduce the symbols for them presently.

'Primate' and 'hairy' in (27) are predicates. 'Predicate' has rather a different meaning in logic from the meaning it typically has in syntax. In syntax, 'predicate' is often roughly synonymous with 'verb'. In logic, however, predicates are terms which represent properties or relations: here, the properties of 'primateness' and 'hairiness'. A logical predicate could thus be a general noun like *primate*, an adjective like *hairy* or a verb like *adore* in *Koko adores the news*. Whereas singular terms refer to specific individuals, predicates refer to general terms, terms which are potentially true of numerous individuals. Being a primate and being hairy are properties which any number of individuals can hold. By contrast, the term *Koko* picks out just a single individual. The properties and relations expressed by predicates can be quite complex and lengthy. For instance, as well as 'is hairy' and 'is a primate', the expressions 'is a good student', 'is taller than the Eiffel tower', 'loves skiing' and 'bought a book on the giant sloth from Amazon' are all predicates. We will discuss these different types of predicate below.

Predicates are typically symbolized by single capital letters. The predicate 'is a primate', for example, could be symbolized P, and the predicate 'is hairy' by H. When expressions containing predicates and singular terms are translated into logical notation, the capitalized predicate symbol is written first, followed by the symbol for the singular term to which the predicate applies. Thus, we can translate the expressions 'Koko is a primate' and 'Koko is hairy' as follows:

(30) Koko is a primate Pk
Koko is hairy Hk

The individual a predicate applies to is called its **argument**: P and H in (30) each have a single argument. But this notation will only get us a certain way. Eventually, we want to be able to translate propositions like 'All primates are hairy'. To do this, we need to examine **quantifiers**. Quantifiers are the logical expressions 'some' and 'all', symbolized by the operators ∃ and ∀ respectively.

Inferences which, like (27) and (29), involve the notions of 'some' and 'all' are very common. Examine the following formula:

(31) (∀x) Px

(31) reads as 'For every x, x is a primate'. What this says is that every individual in the domain in question is a primate. (31) is thus the translation of 'Everything is a primate' (an obviously false statement). Compare this to (32):

(32) (∃x) Px

This reads as 'there exists at least one x, such that x is a primate'. This says that *something* (or someone) is a primate – an obviously true statement.

∀ is known as the **universal quantifier**. Universal quantification is the logical operation which says that a predicate is true of every entity in the domain under discussion. Including ∀ in a formula thus applies the predicate to every entity (argument) in the domain in question. In English, universal quantification can be expressed by the words *all* and *every*, and the phrases *each and every* and *everything*.

∃ is known as the **existential quantifier**. Existential quantification is the logical operation which says that a predicate is true of at least one entity in the domain under discussion. Including ∃ in a formula applies a predicate to at least one entity (argument) in the domain in question. In English, existential quantification can be expressed by the words *some, at least one*, and *something*.

The quantifiers can be combined with the propositional operators. Some examples of this are given below. In (33), the abbreviation S stands for 'is simple', and F stands for 'is fun'.

(33) (∃x) Sx & Fx at least one thing is simple and fun
(∃x) Sx X-OR Fx at least one thing is either simple or fun, but not both
(∃x) ¬Fx something is not fun
(∃x) ¬Sx & ¬Fx something is not simple and not fun
¬(∃x) Fx it's not the case that there is at least one thing that is fun (i.e., nothing is fun)

(∀x) Sx & Fx everything is simple and fun
(∀x) Sx X-OR Fx everything is either simple or fun, but not both
(∀x) ¬Fx everything is not fun (i.e., nothing is fun)
(∀x) ¬Sx & ¬Fx everything is not simple and not fun

The most interesting combinations, however, result from the use of ⊃. Consider the following formula in conjunction with the explanations of the symbols:

(34) P 'is a primate'
H 'is hairy'
$(\forall x)\ Px \supset Hx$

This says that for all x's, if x is a primate then it is hairy. This allows us to give the following translation of the argument in (27), with the justification for the steps shown at the right (k = Koko)

(35) 1. $(\forall x)\ Px \supset Hx$ premise
2. Pk premise
therefore
3. Hk by 1.

'To be hairy' and 'to be a primate' are **one place** predicates: this means that they can only be associated with a single individual constant at a time. (Recall that individual constants, or singular terms, are terms referring to a single individual. Individual constants are sometimes known as **variables**.) For example, the sentence 'Koko and Wilma are primates' can only be expressed logically as (36a), not as (36b).

(36) a. Pk & Pw.
b. P k, w

The formula in (36b) is ill-formed. Since the property of being a primate only ever involves a single individual at a time, one of the constants in (36b) is left 'floating': it is not attached to any predicate, and nothing (even existence) is asserted of it.

Not all predicates are one-place predicates. The predicate 'admire', for example, is a **two-place** predicate: if admiring is going on, then two participants are necessarily involved, the admirer and the admiree. Using A for 'admire', we can express the sentence 'Dietmar admires Horst' as (37) and 'Horst admires Dietmar' as (38):

(37) Ad, h

(38) Ah, d

A two-place predicate can thus be interpreted as indicating a set of ordered pairs of individuals: here, the pair Dietmar and Horst. It is a set of *ordered* pairs precisely because the order in which the individuals occur is crucial: the first individual is the one who admires, the second the one who is admired.

There is no limit on the number of places a predicate may have. 'Give' is an example of a **three-place** predicate, as in G d, b, h 'Dietmar gave the book to Horst'.

We have been defining 'predicate' as a general term expressing a property or a relation. But we may also think of predicates in terms of the individuals to which they apply. Thus, a one-place predicate may be interpreted as a set of individuals: those individuals to which the predicate applies (these are sometimes referred to as the individuals that 'satisfy' the predicate). A two-place predicate applies to an ordered pair of individuals, a three-place predicate to an ordered triple of individuals, and so on. Accordingly, a predicate can have as many places as the members of the ordered n-tuple of individuals that satisfy it.

We are now in a position to be able to produce translations into logical notation of some reasonably complex propositions. These examples involve one- and two-place predicates, and show how the propositional operators are used with them. We first give the logical formula, then a translation into 'logiceeze', then a translation into idiomatic English.

(39) a. $(\forall x)\ Fx \supset Sx$ (S = is simple; F = is fun)
For every x, if x is fun then x is simple
Everything fun is simple.

b. $\neg(\exists x)\ Sx\ \&\ Fx$ (S = is simple; F = is fun)
It is not the case that there is at least one x such that x is simple and x is fun.
Nothing is simple and fun.

c. $(\forall x)\ Tx, l \supset Rx, x$ (T = trusts, R = respects; l = Lucy)
For every x, if x trusts Lucy then x respects x.
Everyone who trusts Lucy respects themselves.

d. $(\forall x)\ Fx \supset Fl$ (F = is fun; l = linguistics)
For every x, if x is fun then linguistics is fun.
If anything is fun then linguistics is fun

e. $(\forall x)\ (Sx\ \&\ \neg Bx) \supset Hx$ (S = is a student; B = is bald; H = is hilarious)
For every x, if x is a student and x is not bald, then x is hilarious.
All students who are not bald are hilarious.

f. $(\exists x)\ Sx\ \&\ (Bx \vee Lx)$ (S = is a student; B = is studying ballet; L =is studying linguistics)
There is at least one x such that x is a student and x is studying ballet or x is studying linguistics.
There is a student who is studying ballet or studying linguistics, or both.

g. $(\forall x)\ (Vx\ \&\ Ix) \supset Ux$ (V = is a virtue; I = is interesting; U = is useful)
For every x, if x is a virtue and x is interesting, then x is useful
All interesting virtues are useful.

h. $(\forall x)\ (Lx\ \&\ Sx) \supset \neg Gx$ (L = is liquid; S = is a substance; G = is a gas)
For every x, if x is a liquid and x is a substance then x is not a gas
Liquid substances are not gases.

i. $(\forall x)\ Vx \supset \neg(Ix \vee Ux)$ (V = is a virtue; I = is interesting; U = is useful)
For every x, if x is a virtue, then it is not the case that x is interesting or x is useful.
No virtue is interesting or useful.

Note that the last example could also be translated as follows

(40) $\neg(\exists x)$ (Vx & (Ix $\vee$ Ux))
It is not the case that there is at least one x, such that x is a virtue and x is interesting or useful.
No virtue is interesting or useful.

The examples given so far involve only a single quantifier. But natural language frequently expresses propositions involving multiple quantification, i.e. expressions which refer to two or more quantities. A two-place predicate, for example, may be quantified in various different ways, some of which we will now illustrate with the two-place predicate R 'remember'.

The simplest case of multiple quantification is where both variables have the same quantifier:

(41) $(\forall x)$ $(\forall y)$ Rx, y (R = remembers)
For every x and for every y it is true that x remembers y.
Everyone remembers everyone.
$(\exists x)$ $(\exists y)$ Rx, y
There is at least one x and at least one y such that x remembers y.
Someone remembers someone.

Note that this formula would be valid in the case where someone remembers themselves.

More complex are cases where one variable receives universal quantification and the other existential. Consider the following example:

(42) $(\exists x)$ $(\forall y)$ Rx, y
There is at least one x such that for every y, x remembers y.
Someone remembers everyone.

Here we will say that $\forall y$ is in the **scope** of $\exists x$. Let's now consider what happens if we swap the order of the individual variables:

(43) $(\exists y)$ $(\forall x)$ Rx, y
There is at least one y such that for every x, x remembers y.
Someone is remembered by everyone.

Here, $\forall x$ is in the scope of $\exists y$. The contrast between (42) and (43) is the difference between an active (42) and a passive (43) sentence. Importantly, the order of the variables in (43) is crucial: (43) is *not* logically equivalent to (44), which expresses a quite different proposition:

(44) $(\forall x)$ $(\exists y)$ Rx, y
For every x, there is at least one y such that x remembers y.
Everyone remembers someone.

The difference between (43) and (44) is subtle but real. (43) says that there is at least one single individual whom everyone remembers. It is the *same* individual who is remembered by everyone: in a universe consisting of Nina, Andrew, Tom, Harry and Briony, Tom might be remembered by Nina, Andrew, Harry and Briony. (44), by contrast, says that every person remembers at least one person. This single person remembered by everybody may well differ from person to person: Briony may remember Harry, Nina may remember Andrew, Andrew may remember Tom. In (44), the existential quantifier is said to be in the scope of the universal quantifier.

To take another example of scope differences, consider the two-place predicate F 'is the father of' in the following two propositions (see Allwood *et al.* 1977: 67 for discussion):

(45) (∀y) (∃x) Fx, y
For every y, there is an x such that x is the father of y.
Everyone has a father.

(46) (∃x) (∀y) Fx, y.
There is at least one x, such that for every y, x is the father of y.
Someone is the father of everyone.

The first proposition, (45), is true, the second, (46), is not. Yet the difference between them consists solely in the order of the existential and universal quantifier, and the consequent scope differences between the two.

Predicate logic notation can be used to precisely represent ambiguities in natural language. Sentence (47a), for example, has, among other readings, (47b) and (47c):

(47) a. Everyone here works for two companies.
b. Everyone works for the same two companies.
c. Everyone works for two companies, which may or may not be the same.

We can represent this difference concisely using the constant p for a person and c for a pair of companies, and the predicate W 'work for':

(48) a. (∃c) (∀p) Wp, c
There is at least one pair of companies c, such that for every person p, p works for c
Everyone works for the same two companies.

b. (∀p) (∃c) Wp, c
For every person p, there is at least one pair of companies c such that p works for c. Everyone works for two companies (which may or may not be the same).

Question Using the abbreviations supplied, (i) translate the following logical formulae into idiomatic English:

P	is a poet	N	is a novelist
T	is talented	W	is a prize winner
S	is a simpleton		

1. $(\exists x)$ Px & Tx
2. $(\forall x)$ Px $\supset$ Sx
3. $(\forall x)$(Px & Wx) $\supset$ Tx
4. $(\exists x)$ Nx & Px

and (ii) translate the following propositions into logical symbolism:

a. No talented novelist is a simpleton.
b. At least one prize-winner is neither talented nor a simpleton.
c. Simpletons are not prize-winners.
d. No talented simpleton is a prize-winning poet.

6.5 Truth, models and extension

For logical approaches to semantics, reference and truth are the principal semantic facts: the most important thing about the meaning of a word is what it refers to, and the most important thing about a sentence is whether or not it is true – whether or not things are as the sentence says they are. Meaning for a logical approach to semantics is thus principally **truth-conditional** (see 3.2.1). As discussed in Chapter 3, for a truth-conditional theory of meaning, knowing the meaning of a factual sentence is the same as knowing what the world would have to be like for that sentence to be true. This does not mean that truth conditions are all there is to meaning. It just means that, as Chierchia and McConnell-Ginet (2000: 72) put it, 'if we ignore the conditions under which S [a sentence] is true, we cannot claim to know the meaning of S. Thus, knowing the truth conditions for S is at least necessary for knowing the meaning of S.'

Logical approaches to semantics deal with the question of truth and reference by providing a **model** for the sets of logical formulae used to represent meaning. The model of a set of logical formulae is a description of a possible world to which the formulae refer, a set of statements showing what each individual constant and predicate refers to in some possible world. The model relates the logical language to this world, by assigning referents to each logical expression. The aim of this is ultimately to produce, for a given set of referents, a statement of the truth values of the logical formulae in which they are included. In other words, the logical formalism will tell us, given a particular world, which sentences describing this world are false and which are true. Given the assumption of the centrality of truth to meaning, this is an important part of describing the meanings of a language. If the logical formulae are identified with sentences of natural language, we will have obtained a logical characterization of the truth conditions of a subset of natural language. We will see a simple example of such a truth-value assignment below.

The referent of a logical expression is called its **extension**. We will consider the extension of both individual constants (singular terms) and of predicates. The extension of an individual constant is simply the individual entity which the constant picks out or refers to in the world. In a universe consisting simply of Tom, Dick, Harry and Jemima, the individual constants *t*, *d*, *h* and *j* have the following extensions: